Problem 12.18

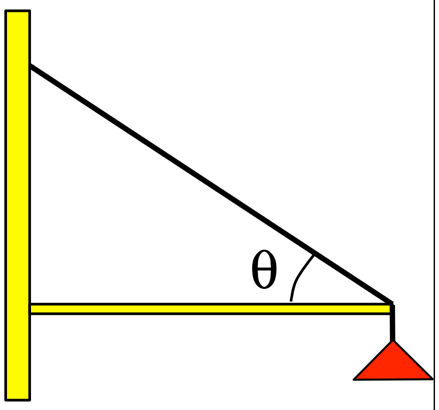
This is another classic *rigid body* problem.

a.) Draw a f.b.d. for the forces acting on the bar.

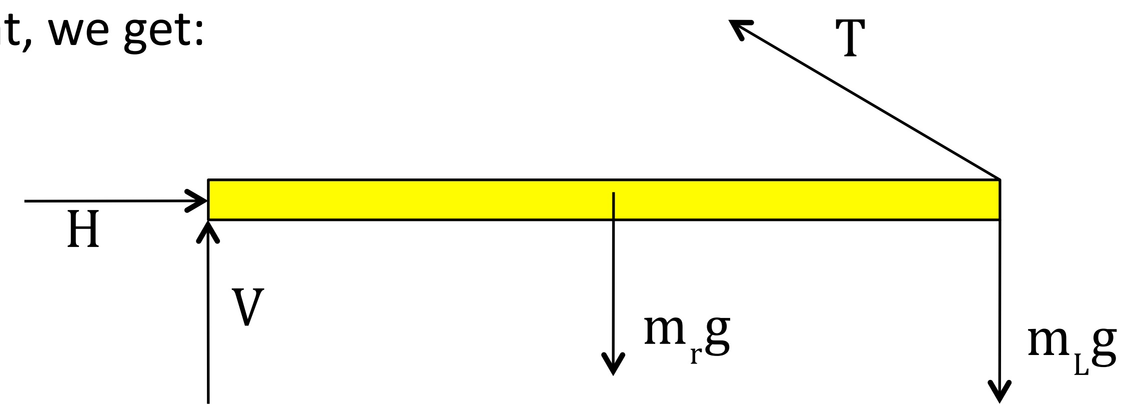
Note 1: Instead of treating the lamp as a separate entity with tension up and gravity down, etc. it is both acceptable and easiest to just identify the lamp's gravitational force $m_L g$ acting downward at the end of the rod.

Note 2: Because we don't know the resultant force at the hinge, we will simply assume horizontal (H) and vertical (V) components acting there.

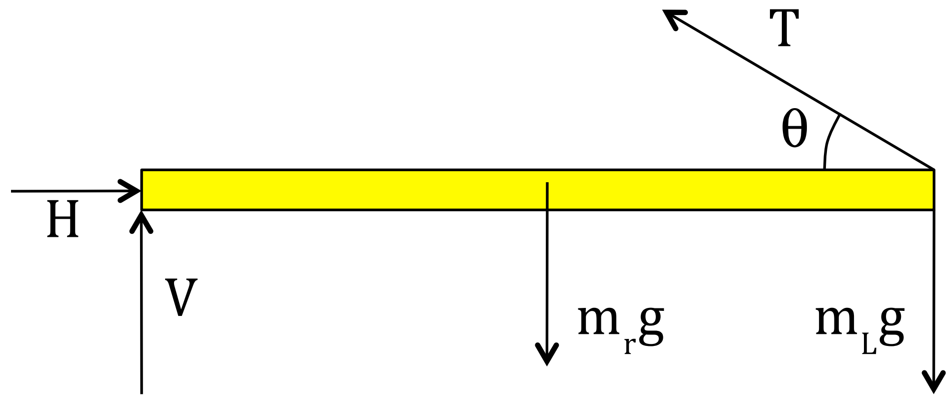
Note 3: The force associated with the mass of the rod can be assumed to be centered at the rod's center of mass.



With all that, we get:



b.) Determine the tension in the cable.

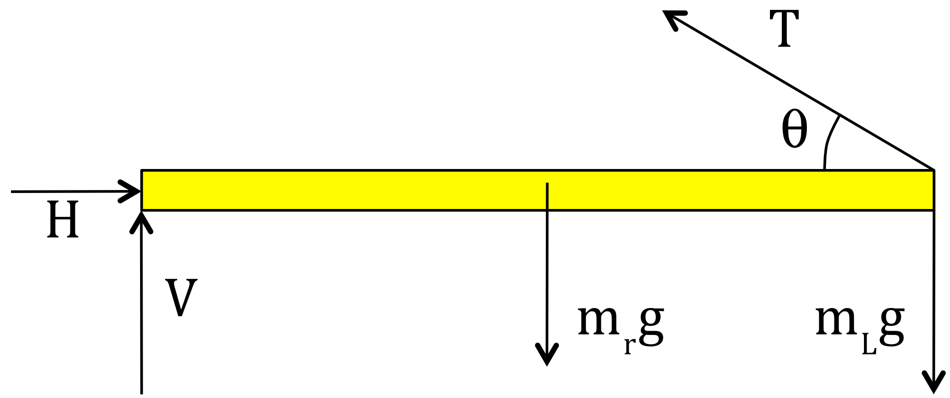


The clever thing to do here is to sum the torques about the pin. Why? Because doing so eliminates the variables *H* and *V* (the torque due to a force acting through the point about which you are taking a torque is zero.

As one other aside, we could use the $r_\perp$ approach to figuring out the torque due to *T*, but if you look at the situation you will realize that the component of *T* that is perpendicular to its $\vec{r}$ (which is in the horizontal) is just the *y-component* of *T*, or $T\sin\theta$. That quantity times *r* (which is just the length of the rod) will give you the magnitude of the torque. (This is just the $F_\perp$ approach.)

Sooo, summing up the torques about the hinge is shown on the next page.

Specifically:



$\underline{\sum \Gamma_{pin}:}$

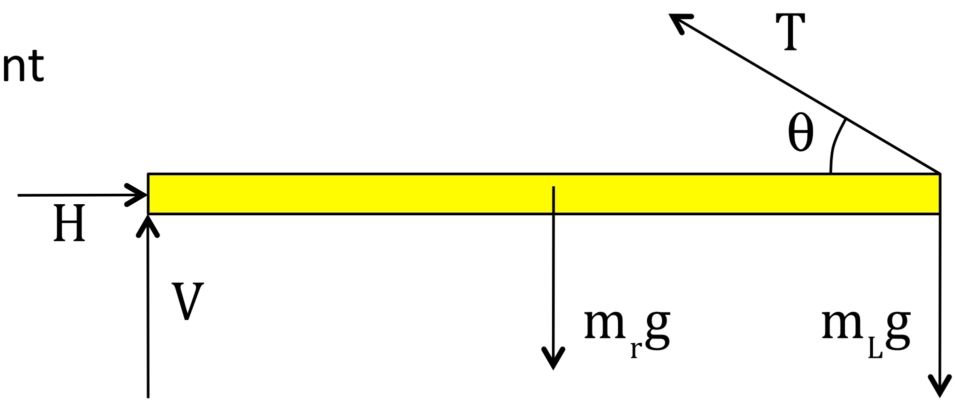
$$-m_r g\left(L/2\right) - m_L g\left(L\right) + \left(T\sin\theta\right)L = I_{floor}\cancel{\alpha}^{\,0}$$

$$\Rightarrow \quad T = \frac{m_r g/2 + m_L g}{\sin\theta}$$

$$= \frac{\left[\left(80.0\text{ N}\right)/2\right]\left(9.80\text{ m/s}^2\right) + \left(20.0\text{ N}\right)\left(9.80\text{ m/s}^2\right)}{\sin 30^\circ}$$

$$= 1176\text{N}$$

c.) Determine the horizontal component of the force acting at hinge.



This is a *sum the forces in the x-direction* problem:

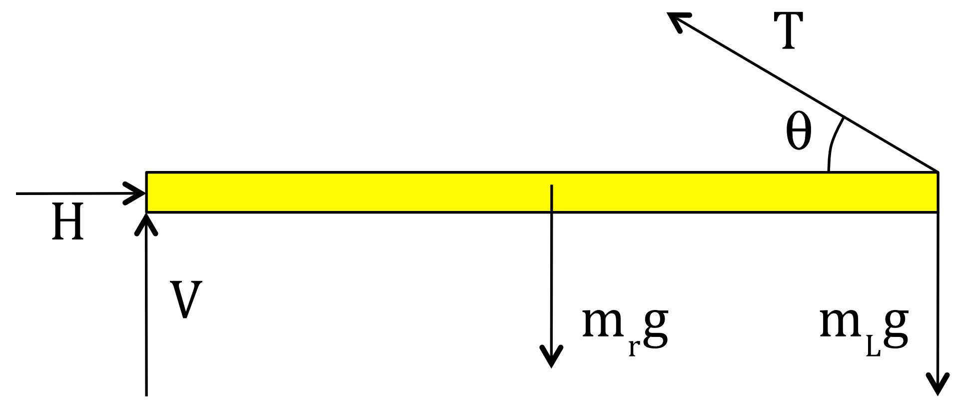
$\underline{\sum F_x:}$

$$H - T\cos\theta = ma_x \quad (a_x = 0)$$

$$\Rightarrow \quad H = (1176\ \text{N})\cos 30^o$$

$$= 1018\ \text{N}$$

d.) Determine the vertical component of the force acting at hinge (or V).



$\sum F_y$:

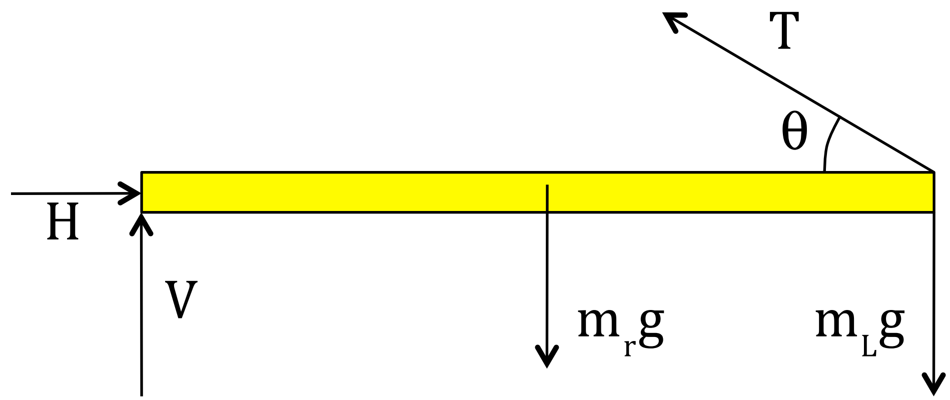
$$V - m_L g - m_r g + T\sin\theta = m\cancel{a_y}^{\,0}$$

$$\Rightarrow \quad V = m_L g + m_r g - T\sin\theta$$

$$= (20.0 \text{ kg})(9.80 \text{ m/s}^2) + (80.0 \text{ kg})(9.80 \text{ m/s}^2) - (1176 \text{ N})\sin 30^{\circ}$$

$$= 392 \text{ N}$$

f.) Determine V by summing torques about lantern's end.



All the forces act through the lantern's rod-connection point except V and m_r, so summing the torques about the lantern yields:

$$\sum \Gamma_{lantern}:$$

$$m_r g\left(L/2\right) - V\left(L\right) = I_{lantern}\cancel{\alpha}^{\,0}$$

$$\Rightarrow \quad V = m_r g / 2$$

$$= \frac{(80.0\text{ N})(9.80\text{ m/s}^2)}{2}$$

$$= 392\text{ N}$$

Note that this is the same value for V that we determined previously.